

# A Decomposition of $m$ -Continuity

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## Abstract

By using an  $m$ -space  $(X, m_X)$ , we define the notions of  $gm$ -closed sets and  $m$ -lc-sets and obtain a decomposition of  $m$ -continuity. Then, the decomposition provides a kind of decomposition of weak forms of continuity.

**Keywords:**  $m$ -structure,  $m$ -space,  $gm$ -closed,  $g$ -closed,  $m$ -lc set, locally closed set, decompositions of weak forms of continuity.

## Introduction

It is known that the notion of decomposition of continuity is important in General Topology. Therefore, many authors [12], [16], [18], [19], [37], [30], [35] and others studied on this subject in General Topology.

In 1970, Levine [21] introduced the notion of generalized closed ( $g$ -closed) sets in topological spaces. Among many modifications of  $g$  closed sets, the notions of  $\alpha g$ -closed [22] (resp.  $gs$ -closed [6],  $gp$ -closed [28],  $\gamma g$ -closed [14],  $gsp$ -closed [9]) sets are investigated by using  $\alpha$ -open (resp. semi-open, preopen,  $b$ -open, semi-preopen) sets.

The present authors [31], [32] introduced and investigated the notions of  $m$ -structures,  $m$ -spaces and  $m$ -continuity. In [27], Noiri introduced the notion of generalized  $m$ -closed ( $gm$ -closed) sets and tried to construct the unified theory of the notions containing  $\alpha g$ -closed sets,  $gs$ -closed sets,  $gp$ -closed sets,  $\gamma g$ -closed sets and  $gsp$ -closed sets.

In this paper, we introduce the notion of  $m$ -lc sets as a modification of locally closed sets. By using the notions of  $gm$ -closed sets and  $m$ -lc sets, we obtain a decomposition of  $m$ -continuity. Then, the decomposition provides a decomposition of weak forms of continuity (semi-continuity, precontinuity,  $\beta$ -continuity etc).

## Preliminaries

Let  $(X, \tau)$  be a topological space and  $A$  a subset of  $X$ . The closure of  $A$  and the interior of  $A$  are denoted by  $Cl(A)$  and  $Int(A)$ , respectively.

**Definition 2.1.** A subset  $A$  of a topological space  $(X, \tau)$  is said to be *semi-open* [20] (resp. *preopen* [24],  *$\alpha$ -open* [26],  *$b$ -open* [4],  *$\beta$ -open* [1] or *semi-preopen* [3]) if  $A \subset \text{Cl}(\text{Int}A)$  (resp.  $A \subset \text{Int}(\text{Cl}(A))$ ,  $A \subset \text{Int}(\text{Cl}(\text{Int}(A)))$ ,  $A \subset \text{Cl}(\text{Int}(A)) \cup \text{Int}(\text{Cl}(A))$ ,  $A \subset \text{Cl}(\text{Int}(\text{Cl}(A)))$ ).

The family of all semi-open (resp. preopen,  $\alpha$ -open,  $b$ -open,  $\beta$ -open) sets in  $(X, \tau)$  is denoted by  $\text{SO}(X)$  (resp.  $\text{PO}(X)$ ,  $\alpha(X)$ ,  $\text{BO}(X)$ ,  $\beta(X)$ ).

**Definition 2.2.** The complement of a semi-open (resp. preopen,  $\alpha$ -open,  $\beta$ -open, semi-preopen,  $b$ -open) set is said to be *semi-closed* [8] (resp. *preclosed* [13],  *$\alpha$ -closed* [25],  *$\beta$ -closed* [1], *semi-preclosed* [3],  *$b$ -closed* [4]).

**Definition 2.3.** The intersection of all semi-closed (resp. preclosed,  $\alpha$ -closed,  $\beta$ -closed, semi-preclosed,  $b$ -closed) sets of  $X$  containing  $A$  is called the *semi-closure* [8] (resp. *preclosure* [13],  *$\alpha$ -closure* [25],  *$\beta$ -closure* [2], *semi-preclosure* [3],  *$b$ -closure* [4]) of  $A$  and is denoted by  $\text{sCl}(A)$  (resp.  $\text{pCl}(A)$ ,  $\alpha\text{Cl}(A)$ ,  $\beta\text{Cl}(A)$ ,  $\text{spCl}(A)$ ,  $\text{bCl}(A)$ ).

**Definition 2.4.** The union of all semi-open (resp. preopen,  $\alpha$ -open,  $\beta$ -open, semi-preopen,  $b$ -open) sets of  $X$  contained in  $A$  is called the *semi-interior* (resp. *preinterior*,  *$\alpha$ -interior*,  *$\beta$ -interior*, *semi-preinterior*,  *$b$ -interior*) of  $A$  and is denoted by  $\text{sInt}(A)$  (resp.  $\text{pInt}(A)$ ,  $\alpha\text{Int}(A)$ ,  $\beta\text{Int}(A)$ ,  $\text{spInt}(A)$ ,  $\text{bInt}(A)$ ).

**Definition 2.5.** Let  $(X, \tau)$  be a topological space. A subset  $A$  of  $X$  is said to be

- (1)  *$g$ -closed* [21] if  $\text{Cl}(A) \subset U$  whenever  $A \subset U$  and  $U \in \tau$ ,
- (2)  *$\alpha g$ -closed* [22] if  $\alpha\text{Cl}(A) \subset U$  whenever  $A \subset U$  and  $U \in \tau$ ,
- (3)  *$gs$ -closed* [6] if  $\text{sCl}(A) \subset U$  whenever  $A \subset U$  and  $U \in \tau$ ,
- (4)  *$gp$ -closed* [28] if  $\text{pCl}(A) \subset U$  whenever  $A \subset U$  and  $U \in \tau$ ,
- (5)  *$gb$ -closed* (or  *$\gamma g$ -closed* [14]) if  $\text{bCl}(A) \subset U$  whenever  $A \subset U$  and  $U \in \tau$ ,
- (6)  *$gsp$ -closed* (or  *$g\beta$ -closed*) [9] if  $\text{spCl}(A) \subset U$  whenever  $A \subset U$  and  $U \in \tau$ .

**Definition 2.6.** Let  $(X, \tau)$  be a topological space. A subset  $A$  is called a *locally closed* set (briefly *LC-set*) [7], [15] (resp.  *$B$ -set* [36],  *$A_7$ -set* [37],  *$\eta$ -set* [30],  *$BC$ -set* [19],  *$C$ -set* [18]) if  $A = U \cap F$ , where  $U$  is open and  $F$  is closed (resp. semi-closed, preclosed,  $\alpha$ -closed,  $b$ -closed, semi-preclosed).

Throughout the present paper,  $(X, \tau)$  and  $(Y, \sigma)$  always denote topological spaces and  $f : (X, \tau) \rightarrow (Y, \sigma)$  presents a function.

**Definition 2.7.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be *semi-continuous* [20] (resp. *precontinuous* [24],  *$\alpha$ -continuous* [25],  *$b$ -continuous* [4],  *$\beta$ -continuous* [1]) if  $f^{-1}(V)$  is a semi-open (resp. preopen,  $\alpha$ -open,  $b$ -open,  $\beta$ -open) set in  $(X, \tau)$  for each open set  $V$  of  $(Y, \sigma)$ .

**Definition 2.8.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be  *$g$ -continuous* [5] (resp.  *$gs$ -continuous* [11],  *$gp$ -continuous* [28],  *$\alpha g$ -continuous* [22],  *$\gamma g$ -continuous* [14],  *$gsp$ -continuous* [9]) if  $f^{-1}(F)$  is  *$g$ -closed* (resp.  *$gs$ -closed*,  *$gp$ -closed*,  *$\alpha g$ -closed*,  *$\gamma g$ -closed*,  *$gsp$ -closed*) in  $(X, \tau)$  for every closed set  $F$  of  $(Y, \sigma)$ .

## $m$ -Continuity

**Definition 3.1.** A subfamily  $m_X$  of the power set  $\mathcal{P}(X)$  of a nonempty set  $X$  is called a *minimal structure* (briefly  *$m$ -structure*) [31], [32] on  $X$  if  $\emptyset \in m_X$  and  $X \in m_X$ .

By  $(X, m_X)$ , we denote a nonempty set  $X$  with a minimal structure  $m_X$  on  $X$  and call it an  *$m$ -space*. Each member of  $m_X$  is said to be  *$m_X$ -open* and the complement of a  $m_X$ -open set is said to be  *$m_X$ -closed*.

**Remark 3.1.** Let  $(X, \tau)$  be a topological space. Then the families  $\tau$ ,  $\text{SO}(X)$ ,  $\text{PO}(X)$ ,  $\alpha(X)$ ,  $\text{BO}(X)$  and  $\beta(X)$  are all  $m$ -structures on  $X$ .

**Definition 3.2.** Let  $X$  be a nonempty set and  $m_X$  an  $m$ -structure on  $X$ . For a subset  $A$  of  $X$ , the  $m_X$ -closure of  $A$  and the  $m_X$ -interior of  $A$  are defined in [23] as follows:

- (1)  $m_X\text{-Cl}(A) = \cap\{F : A \subset F, X - F \in m_X\}$ ,
- (2)  $m_X\text{-Int}(A) = \cup\{U : U \subset A, U \in m_X\}$ .

**Remark 3.2.** Let  $(X, \tau)$  be a topological space and  $A$  be a subset of  $X$ . If  $m_X = \tau$  (resp.  $\text{SO}(X)$ ,  $\text{PO}(X)$ ,  $\alpha(X)$ ,  $\beta(X)$ ,  $\text{BO}(X)$ ), then we have

- (1)  $m_X\text{-Cl}(A) = \text{Cl}(A)$  (resp.  $s\text{Cl}(A)$ ,  $p\text{Cl}(A)$ ,  $\alpha\text{Cl}(A)$ ,  $\beta\text{Cl}(A)$ ,  $b\text{Cl}(A)$ ),
- (2)  $m_X\text{-Int}(A) = \text{Int}(A)$  (resp.  $s\text{Int}(A)$ ,  $p\text{Int}(A)$ ,  $\alpha\text{Int}(A)$ ,  $\beta\text{Int}(A)$ ,  $b\text{Int}(A)$ ).

**Lemma 3.1** (Maki et al. [23]). Let  $(X, m_X)$  be an  $m$ -space. For subsets  $A$  and  $B$  of  $X$ , the following properties hold:

- (1)  $m_X\text{-Cl}(X - A) = X - m_X\text{-Int}(A)$  and  $m_X\text{-Int}(X - A) = X - m_X\text{-Cl}(A)$ ,
- (2) If  $(X - A) \in m_X$ , then  $m_X\text{-Cl}(A) = A$  and if  $A \in m_X$ , then  $m_X\text{-Int}(A) = A$ ,
- (3)  $m_X\text{-Cl}(\emptyset) = \emptyset$ ,  $m_X\text{-Cl}(X) = X$ ,  $m_X\text{-Int}(\emptyset) = \emptyset$  and  $m_X\text{-Int}(X) = X$ ,
- (4) If  $A \subset B$ , then  $m_X\text{-Cl}(A) \subset m_X\text{-Cl}(B)$  and  $m_X\text{-Int}(A) \subset m_X\text{-Int}(B)$ ,
- (5)  $A \subset m_X\text{-Cl}(A)$  and  $m_X\text{-Int}(A) \subset A$ ,
- (6)  $m_X\text{-Cl}(m_X\text{-Cl}(A)) = m_X\text{-Cl}(A)$  and  $m_X\text{-Int}(m_X\text{-Int}(A)) = m_X\text{-Int}(A)$ .

**Definition 3.3.** A minimal structure  $m_X$  on a nonempty set  $X$  is said to have *property  $\mathcal{B}$*  [23] if the union of any family of subsets belonging to  $m_X$  belongs to  $m_X$ .

**Remark 3.3.** Let  $(X, \tau)$  be a topological space and  $m_X = \text{SO}(X)$  (resp.  $\text{PO}(X)$ ,  $\alpha(X)$ ,  $\beta(X)$ ,  $\text{BO}(X)$ ), then  $m_X$  satisfies property  $\mathcal{B}$ .

**Lemma 3.2** (Popa and Noiri [33]). Let  $(X, m_X)$  be an  $m$ -space and  $m_X$  satisfies property  $\mathcal{B}$ . Then for a subset  $A$  of  $X$ , the following properties hold:

- (1)  $A \in m_X$  if and only if  $m_X\text{-Int}(A) = A$ ,
- (2)  $A$  is  $m_X$ -closed if and only if  $m_X\text{-Cl}(A) = A$ ,
- (3)  $m_X\text{-Int}(A) \in m_X$  and  $m_X\text{-Cl}(A)$  is  $m_X$ -closed.

**Definition 3.4.** Let  $(X, \tau)$  be a topological space and  $m_X$  an  $m$ -structure on  $X$ . A subset  $A$  is said to be *generalized  $m$ -closed* (briefly *gm-closed*) [27] if  $m_X\text{-Cl}(A) \subset U$  whenever  $A \subset U$  and  $U \in \tau$ . The complement of a *gm-closed* set is said to be *gm-open*.

**Remark 3.4.** Let  $(X, \tau)$  be a topological space and  $A$  a subset of  $X$ . If  $m_X = \tau$  (resp.  $\text{SO}(X)$ ,  $\text{PO}(X)$ ,  $\alpha(X)$ ,  $\text{BO}(X)$ ,  $\beta(X)$ ) and  $A$  is *gm-closed*, then  $A$  is *g-closed* (resp. *gs-closed*, *gp-closed*, *ag-closed*, *γg-closed*, *gsp-closed*).

**Definition 3.5.** Let  $(X, \tau)$  be a topological space and  $m_X$  an  $m$ -structure on  $X$ . A subset  $A$  is called an  *$m$ -lc set* if  $A = U \cap F$ , where  $U \in \tau$  and  $F$  is  $m_X$ -closed.

**Remark 3.5.** Let  $(X, \tau)$  be a topological space and  $A$  a subset of  $X$ . If  $m_X = \tau$  (resp.  $\text{SO}(X)$ ,  $\text{PO}(X)$ ,  $\alpha(X)$ ,  $\text{BO}(X)$ ,  $\beta(X)$ ) and  $A$  is an  *$m$ -lc set*, then  $A$  is an *LC set* (resp. a *B-set*, an  *$A_7$ -set*, an  *$\eta$ -set*, a *BC-set*, a *C-set*).

**Definition 3.6.** Let  $f : X \rightarrow Y$  be a function, where  $X$  is a nonempty set with a minimal structure  $m_X$  and  $Y$  is a topological space. The function  $f : X \rightarrow Y$  is said to be  *$m$ -continuous* [32] if for

each  $x \in X$  and each open set  $V$  of  $Y$  containing  $f(x)$ , there exists a subset  $U \in m_X$  containing  $x$  such that  $f(U) \subset V$ .

**Lemma 3.3** (Popa and Noiri [32]). For a function  $f : X \rightarrow Y$ , where  $X$  is a nonempty set with a minimal structure  $m_X$  and  $Y$  is a topological space, the following properties are equivalent:

- (1)  $f$  is  $m$ -continuous;
- (2)  $f^{-1}(V) = m_X\text{-Int}(f^{-1}(V))$  for every open set  $V$  of  $Y$ ;
- (3)  $m_X\text{-Cl}(f^{-1}(F)) = f^{-1}(F)$  for every closed set  $F$  of  $Y$ .

**Corollary 3.1.** (Popa and Noiri [32]) Let  $X$  be a nonempty set with a minimal structure  $m_X$  satisfying property  $\mathcal{B}$  and  $Y$  a topological space. For a function  $f : X \rightarrow Y$ , the following are equivalent:

- (1)  $f$  is  $m$ -continuous;
- (2)  $f^{-1}(V)$  is  $m_X$ -open in  $(X, m_X)$  for every open set  $V$  of  $Y$ ;
- (3)  $f^{-1}(F)$  is  $m_X$ -closed in  $(X, m_X)$  for every closed set  $F$  of  $Y$ .

**Remark 3.6.** Let  $(X, \tau)$  be a topological space and  $m_X$  an  $m$ -structure on  $X$ . If  $f : (X, \tau) \rightarrow (Y, \sigma)$  is  $m$ -continuous and  $m_X = \tau$  (resp.  $\text{SO}(X)$ ,  $\text{PO}(X)$ ,  $\alpha(X)$ ,  $\text{BO}(X)$ ,  $\beta(X)$ ), then  $f$  is continuous (resp. semi-continuous, precontinuous,  $\alpha$ -continuous,  $b$ -continuous,  $\beta$ -continuous).

## Decompositions of $m$ -continuity

**Theorem 4.1.** Let  $(X, \tau)$  be a topological space and  $m_X$  a minimal structure on  $X$  having property  $\mathcal{B}$ . Then a subset  $A$  of  $X$  is  $m_X$ -closed if and only if it is  $gm$ -closed and an  $m$ -lc set.

**Proof.** Necessity: Suppose that  $A$  is  $m_X$ -closed in  $X$ . Let  $A \subset U$  and  $U \in \tau$ . Since  $A$  is  $m_X$ -closed, by Lemma 3.2  $A = m_X\text{-Cl}(A)$  and hence  $m_X\text{-Cl}(A) \subset U$ . Therefore,  $A$  is  $gm$ -closed. Since  $A = X \cap A$ ,  $A$  is an  $m$ -lc set.

Sufficiency: Suppose that  $A$  is  $gm$ -closed and an  $m$ -lc set. Since  $A$  is an  $m$ -lc set,  $A = U \cap F$ , where  $U \in \tau$  and  $F$  is  $m_X$ -closed in  $X$ . Therefore, we have  $A \subset U$  and  $A \subset F$ . By the hypothesis, we obtain  $m_X\text{-Cl}(A) \subset U$  and  $m_X\text{-Cl}(A) \subset F$  and hence  $m_X\text{-Cl}(A) \subset U \cap F = A$ . Thus,  $m_X\text{-Cl}(A) = A$  and by Lemma 3.2  $A$  is  $m_X$ -closed.

**Corollary 4.1.** Let  $A$  be a subset of a topological space  $(X, \tau)$ . Then, the following properties hold:

- (1)  $A$  is closed if and only if  $A$  is  $g$ -closed and an  $LC$ -set.
- (2)  $A$  is semi-closed if and only if  $A$  is  $gs$ -closed and a  $B$ -set.
- (3)  $A$  is pre-closed if and only if  $A$  is  $gp$ -closed and an  $A_7$ -set.
- (4)  $A$  is  $\alpha$ -closed if and only if  $A$  is  $\alpha g$ -closed and an  $\eta$ -set.
- (5)  $A$  is  $b$ -closed if and only if  $A$  is  $\gamma g$ -closed and a  $BC$ -set.
- (6)  $A$  is  $\beta$ -closed if and only if  $A$  is  $gsp$ -closed and a  $C$ -set.

**Definition 4.1.** Let  $(X, \tau)$  be a topological space and  $m_X$  a minimal structure on  $X$ . A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be

- (1)  $gm$ -continuous if  $f^{-1}(F)$  is  $gm$ -closed in  $(X, \tau)$  for each closed set  $F$  of  $(Y, \sigma)$ ,
- (2) contra  $m$ -lc-continuous if  $f^{-1}(F)$  is an  $m$ -lc set of  $(X, \tau)$  for each closed set  $F$  of  $(Y, \sigma)$ .

**Remark 4.1.** Let  $(X, \tau)$  be a topological space and  $m_X$  an  $m$ -structure on  $X$ .

(1) If  $m_X = \tau$  (resp.  $\text{SO}(X)$ ,  $\text{PO}(X)$ ,  $\alpha(X)$ ,  $\text{BO}(X)$ ,  $\beta(X)$ ) and  $f : (X, \tau) \rightarrow (Y, \sigma)$  is  $gm$ -continuous, then we obtain Definition 2.8.

(2) If  $m_X = \tau$  (resp.  $\text{SO}(X)$ ,  $\text{PO}(X)$ ,  $\alpha(X)$ ,  $\text{BO}(X)$ ,  $\beta(X)$ ) and  $f : (X, \tau) \rightarrow (Y, \sigma)$  is contra  $m$ -lc-continuous, then  $f$  is said to be *contra LC-continuous* (resp. *contra B-continuous*, *contra  $A_\tau$ -continuous*, *contra  $\eta$ -continuous*, *contra BC-continuous*, *contra C-continuous*).

**Theorem 4.2.** Let  $(X, \tau)$  be a topological space and  $m_X$  a minimal structure on  $X$  having property  $\mathcal{B}$ . Then a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is  $m$ -continuous if and only if  $f$  is  $gm$ -continuous and contra  $m$ -lc-continuous.

**Proof.** This is an immediate consequence of Theorem 4.1 and Corollary 3.1.

**Corollary 4.2.** For a function  $f : (X, \tau) \rightarrow (Y, \sigma)$ , the following properties hold:

- (1)  $f$  is continuous if and only if  $f$  is  $g$ -continuous and contra  $LC$ -continuous.
- (2)  $f$  is semi-continuous if and only if  $f$  is  $gs$ -continuous and contra  $B$ -continuous.
- (3)  $f$  is precontinuous if and only if  $f$  is  $gp$ -continuous and contra  $A_\tau$ -continuous. (4)  $f$  is  $\alpha$ -continuous if and only if  $f$  is  $\alpha g$ -continuous and contra  $\eta$ -continuous. (5)  $f$  is  $\gamma g$ -continuous if and only if  $f$  is  $\gamma g$ -continuous and contra  $BC$ -continuous. (6)  $f$  is  $\beta$ -continuous if and only if  $f$  is  $gsp$ -continuous and contra  $C$ -continuous.

**Definition 4.2.** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be *contra-continuous* [10] if  $f^{-1}(F)$  is open in  $(X, \tau)$  for each closed set  $F$  of  $(Y, \sigma)$ .

**Theorem 4.3.** Let  $(X, \tau)$  be a topological space and  $m_X$  a minimal structure on  $X$  having property  $\mathcal{B}$ . Then, a contra continuous function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is  $m$ -continuous if and only if  $f$  is  $gm$ -continuous.

**Proof.** Suppose that  $f$  is contra continuous and  $gm$ -continuous. Let  $F$  be any closed set of  $(Y, \sigma)$ . Since  $f$  is contra-continuous,  $f^{-1}(F)$  is open in  $(X, \tau)$  and hence an  $m$ -lc-set of  $(X, \tau)$ . Since  $f$  is  $gm$ -continuous,  $f^{-1}(F)$  is  $gm$ -closed and hence, by Theorem 4.1,  $f^{-1}(F)$  is  $m$ -closed. Therefore,  $f$  is  $m$ -continuous. The converse is obvious.

**Corollary 4.3.** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a contra-continuous function. Then the following properties hold:

- (1)  $f$  is continuous if and only if  $f$  is  $g$ -continuous.
- (2)  $f$  is semi-continuous if and only if  $f$  is  $gs$ -continuous.
- (3)  $f$  is pre-continuous if and only if  $gp$ -continuous.
- (4)  $f$  is  $\alpha$ -continuous if and only if  $f$  is  $\alpha g$ -continuous.
- (5)  $f$  is  $b$ -continuous if and only if  $f$  is  $\gamma g$ -continuous. (6)  $f$  is  $\beta$ -continuous if and only if  $f$  is  $gsp$ -continuous.

## New forms of decomposition of $m$ -continuity

First, we recall the  $\theta$ -closure and the  $\delta$ -closure of a subset in a topological space. Let  $(X, \tau)$  be a topological space and  $A$  a subset of  $X$ . A point  $x \in X$  is called a  $\theta$ -cluster (resp.  $\delta$ -cluster) point of  $A$  if  $\text{Cl}(V) \cap A \neq \emptyset$  (resp.  $\text{Int}(\text{Cl}(V)) \cap A \neq \emptyset$ ) for every open set  $V$  containing  $x$ . The set of all  $\theta$ -cluster (resp.  $\delta$ -cluster) points of  $A$  is called the  $\theta$ -closure (resp.  $\delta$ -closure) of  $A$  and is denoted by  $\text{Cl}_\theta(A)$  (resp.  $\text{Cl}_\delta(A)$ )[38].

**Definition 5.1.** A subset  $A$  of a topological space  $(X, \tau)$  is said to be (1)  $\delta$ -preopen [34] (resp.  $\theta$ -preopen [29]) if  $A \subset \text{Int}(\text{Cl}_\delta(A))$  (resp.  $A \subset \text{Int}(\text{Cl}_\theta(A))$ ),

(2)  $\delta$ - $\beta$ -open [17](resp.  $\theta$ - $\beta$ -open [29]) if  $A \subset \text{Cl}(\text{Int}(\text{Cl}_\delta(A)))$  (resp.  $A \subset \text{Cl}(\text{Int}(\text{Cl}_\theta(A)))$ ).

By  $\delta\text{PO}(X)$  (resp.  $\delta\beta(X)$ ,  $\theta\text{PO}(X)$ ,  $\theta\beta(X)$ ), we denote the collection of all  $\delta$ -preopen (resp.  $\delta$ - $\beta$ -open,  $\theta$ -preopen,  $\theta$ - $\beta$ -open) sets of a topological space  $(X, \tau)$ . These four collections are  $m$ -structures with property  $\mathcal{B}$ .

**Definition 5.2.** The complement of a  $\delta$ -preopen (resp.  $\theta$ -preopen,  $\delta$ - $\beta$ -open,  $\theta$ - $\beta$ -open) set is said to be  $\delta$ -preclosed (resp.  $\theta$ -preclosed,  $\delta$ - $\beta$ -closed,  $\theta$ - $\beta$ -closed).

**Definition 5.3.** Let  $(X, \tau)$  be a topological space and  $A$  a subset of  $X$ . The intersection of all  $\delta$ -preclosed (resp.  $\theta$ -preclosed,  $\delta$ - $\beta$ -closed,  $\theta$ - $\beta$ -closed) sets of  $X$  containing  $A$  is called the  $\delta$ -preclosure (resp.  $\theta$ -preclosure,  $\delta$ - $\beta$ -closure,  $\theta$ - $\beta$ -closure) of  $A$  and is denoted by  $\text{pCl}_\delta(A)$  (resp.  $\text{pCl}_\theta(A)$ ,  $\text{spCl}_\delta(A)$ ,  $\text{spCl}_\theta(A)$ ).

For subsets of a topological space  $(X, \tau)$ , we can define many new variations of  $g$ -closed sets. For example, in case  $m_X = \delta\text{PO}(X)$ ,  $\delta\beta(X)$ ,  $\theta\text{PO}(X)$ ,  $\theta\beta(X)$ , we can define new types of  $g$ -closed sets as follows:

**Definition 5.4.** A subset  $A$  of a topological space  $(X, \tau)$  is said to be  $g\delta p$ -closed [19] (resp.  $g\theta p$ -closed,  $g\delta sp$ -closed,  $g\theta sp$ -closed) if  $\text{Cl}(A) \subset U$  whenever  $A \subset U$  and  $U$  is  $\delta$ -preopen (resp.  $\theta$ -preopen,  $\delta$ - $\beta$ -open,  $\theta$ - $\beta$ -open) in  $(X, \tau)$ .

**Definition 5.5.** A subset  $A$  of a topological space  $(X, \tau)$  is called a  $\delta p$ -lc set or  $\xi$ -set [19] (resp.  $\theta p$ -lc set,  $\delta\beta$ -lc set,  $\theta\beta$ -lc set) if  $A = U \cap F$ , where  $U$  is open in  $(X, \tau)$  and  $F$  is  $\delta p$ -closed (resp.  $\theta p$ -closed,  $\delta$ - $\beta$ -closed,  $\theta$ - $\beta$ -closed) in  $(X, \tau)$ .

**Corollary 5.1.** For a subset  $A$  of a topological space  $(X, \tau)$ , the following properties hold:

- (1)  $A$  is  $\delta$ -preclosed if and only if  $A$  is  $g\delta p$ -closed and a  $\delta p$ -lc set (Theorem 4 of [19]).
- (2)  $A$  is  $\theta$ -preclosed if and only if  $A$  is  $g\theta p$ -closed and a  $\theta p$ -lc set.
- (3)  $A$  is  $\delta$ - $\beta$ -closed if and only if  $A$  is  $g\delta sp$ -closed and a  $\delta\beta$ -lc set.
- (4)  $A$  is  $\theta$ - $\beta$ -closed if and only if  $A$  is  $g\theta sp$ -closed and a  $\theta\beta$ -lc set.

**Proof.** Let  $m_X = \delta\text{PO}(X)$ ,  $\theta\text{PO}(X)$ ,  $\delta\beta(X)$  and  $\theta\beta(X)$ . Then this is an immediate consequence of Theorem 4.1.

By defining functions similarly to Definition 4.1, we obtain the following decompositions of weak forms of continuity:

**Corollary 5.2.** For a function  $f : (X, \tau) \rightarrow (Y, \sigma)$ , the following properties hold:

- (1)  $f$  is  $\delta$ -precontinuous if and only if  $f$  is  $g\delta p$ -continuous and  $\delta plc$ -continuous.
- (2)  $f$  is  $\theta$ -precontinuous if and only if  $f$  is  $g\theta p$ -continuous and  $\theta plc$ -continuous.
- (3)  $f$  is  $\delta$ - $\beta$ -continuous if and only if  $f$  is  $g\delta sp$ -continuous and  $\delta\beta$ -lc-continuous.
- (4)  $f$  is  $\theta$ - $\beta$ -continuous if and only if  $f$  is  $g\theta sp$ -continuous and  $\theta\beta$ -lc-continuous.

**Proof.** This is an immediate consequence of Theorem 4.2.

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## O descompunere a $m$ -continuității

### Rezumat

Folosind un  $m$ -spațiu  $(X, m_X)$ , definim noțiunile de mulțimi  $gm$ -încise și de  $m$ -lc-mulțimi și obținem o descompunere a  $m$ -continuității. Această descompunere permite apoi obținerea unor descompuneri ale formelor slabe de continuitate.